Topology

B. Math. II

Semestral Examination

Instructions: All questions carry equal marks.

1. Show that if a topological space X has a countable basis $\{B_n\}$, then every basis $\mathcal{C}$ of X contains a countable basis for X.
2. Define Hausdorff topological space. Show that X is Hausdorff if and only if the **diagonal** $\Delta = \{x \times x \mid x \in X\}$ is closed in $X \times X$.
3. Define connected topological space. Let A and B be proper subsets of connected spaces X and Y respectively. Prove that the complement of $A \times B$ in $X \times Y$ is connected.
4. Define a path connected space. Prove that if U is an open connected subspace of $\mathbb{R}^2$, then U is path connected. Is the result also true for closed subspaces of $\mathbb{R}^2$? Justify your answer.
5. If Y is a compact space, then prove that for any space X, the projection map $\pi_1 : X \times Y \to X$ is a closed map.
6. Define normal topological space. Prove that every compact Hausdorff space is normal. Is the converse true? Justify your answer.